

Capital and Income Inequality: an Aggregate-Demand Complementarity

Florin O. Bilbiie, Diego R. Känzig and Paolo Surico

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- **Capital** is **central** to macro: growth, inequality, optimal taxation, business cycles
- But for **monetary transmission mechanism**, capital traditionally plays **side show**
 - Workhorse model: New Keynesian (NK): **No K?**
 - Ingredient in DSGE versions, but back seat for **consumption**
- Growing literature on **heterogeneous agent** NK models (TANK & HANK)
 - *What role does capital play in these models?*

Our Results

- **Heterogeneity** puts **capital** front and center in NK
- Isolate two **amplification channels** for monetary policy:
 - Cyclical income inequality (Bilbiie, 2008)
 - Unequal capital expenditures (**this paper**)
- Show that they are **complementary**

Effects of monetary policy on consumption		
	No income inequality	Income Inequality
No capital inequality	<i>representative agent</i>	small
Capital inequality	small	LARGE

- Joint effect *much larger* than individual effects combined

- Role of **redistribution**: key which income
 - **Opposite effects** of redistributing **capital income** vs. **profits**
 - Cyclicalities of income matters!
- Complementarity **robust** to adding wage rigidity, idiosyncratic risk, ...
- Novel analytics for tractable HANK model with capital (and RANK)

Related Literature

- **Empirical:** Campbell and Mankiw (1989); Kaplan, Violante, and Weidner (2014); Cloyne, Ferreira, and Surico (2020); Coibion et al. (2017)
- **RANK with capital:** Dupor (2001); Sveen and Weinke (2005); Woodford (2005); Rupert and Šustek (2019)
- **2000s TANK:** Galí, López-Salido, and Vallés (2007); Bilbiie (2008)
- **2010s HANK with focus on C:** Kaplan, Moll, and Violante (2018); Gornemann, Kuester, and Nakajima (2016); Bayer et al. (2019); Luetticke (forthcoming); Auclert (2019); Auclert and Rognlie (2018); Debortoli and Galí (2018); Guerrieri and Lorenzoni (2017); McKay, Nakamura, and Steinsson (2016); Challe et al. (2017); Hagedorn et al. (2019); Werning (2015); Ravn and Sterk (2020); Cui and Sterk (2019); Bilbiie (2018) ...
- **Now: HANK with focus on K:** Auclert, Rognlie, and Straub (2020); Alves et al. (2019); **this paper**

A Tale of Two Inequalities

Generic **budget constraint**

$$C^j + S^j = Y^j$$

Two inequalities

- **Capital inequality**: unequal savings/capital expenditure (LHS)

$$C^j + S^j = Y$$

- **Income inequality**: unequal incomes, e.g. labor vs. financial (RHS)

$$C^j = Y^j$$

A Simple Model

- $1 - \lambda$ **savers** S with bonds Euler

$$c_t^S = E_t c_{t+1}^S - r_t,$$

isoelastic *investment*

$$i_t = \eta y_t,$$

and budget constraint

$$C_Y c_t^S + \frac{I_Y}{1 - \lambda} i_t = Y_Y^S y_t^S$$

- λ **hand-to-mouth** H :

$$c_t^H = y_t^H = \chi y_t,$$

where χ is an *income distribution* model

Isolating Capital Inequality

- Assume that income is **perfectly redistributed** $\chi = 1$

$$C_Y c_t^S + \frac{I_Y}{1-\lambda} i_t = Y_Y^S y_t$$
$$c_t^H = y_t$$

- Aggregate Euler

$$c_t = E_t c_{t+1} - \underbrace{\frac{1-\lambda}{1-\lambda \frac{1-I_Y}{1-\eta I_Y}}}_{\text{Multiplier (= 1 in RA)}} r_t$$

- Keynesian-cross style **multiplier** if $\eta > 1$

\Rightarrow the **savings rate** (of S) acts as an **MPC** (of H)

- S investment \rightarrow Capital income, redistribution $\rightarrow H$ consumption \uparrow
- Channel likely to operate in any HA model with some net saving, micro-foundation of Samuelson (1939)'s famous multiplier-accelerator

- Increasing in λ as long as $\lambda \frac{1-I_Y}{1-\eta I_Y} < 1$

Isolating Income Inequality

- Assume that there is **no capital investment** $I_Y = 0$

$$c_t^S = \frac{1 - \lambda\chi}{1 - \lambda} y_t$$

$$c_t^H = \chi y_t$$

- Aggregate Euler

$$c_t = E_t c_{t+1} - \underbrace{\frac{1 - \lambda}{1 - \lambda\chi}}_{\text{Multiplier (= 1 in RA)}} r_t$$

- Another Keynesian-cross style **multiplier if $\chi > 1$** : **countercyclical inequality**
 - $\Delta AD \rightarrow \Delta Y \rightarrow \Delta \text{demand of } H \rightarrow \Delta AD$
 - Well understood (Bilbiie, 2008, 2018; Auclert, 2019; Patterson, 2019) and present in many HA models
 - Increasing in λ as long as $\lambda\chi < 1$

The Multiplier ... of the Multiplier

- Allow for **capital** and **income inequality**

$$\left| \frac{\partial c_t}{\partial r_t} \right| = \frac{1 - \lambda}{1 - \lambda \chi \frac{1 - l_Y}{1 - \eta l_Y}}$$

- **Complementarity** iff income inequality is countercyclical $\chi > 1$ and investment is more than one-to-one procyclical $\eta > 1$:

$$\left| \frac{\partial c_t}{\partial r_t} \right|_{K, \text{ no redistrib}} > \left| \frac{\partial c_t}{\partial r_t} \right|_{\text{no } K, \text{ no redistrib}} \times \left| \frac{\partial c_t}{\partial r_t} \right|_{K, \text{ redistrib}}.$$

Multiplier (Samuelson, 1948)

$$\frac{1}{1 - x}$$

- x is the aggregate MPC

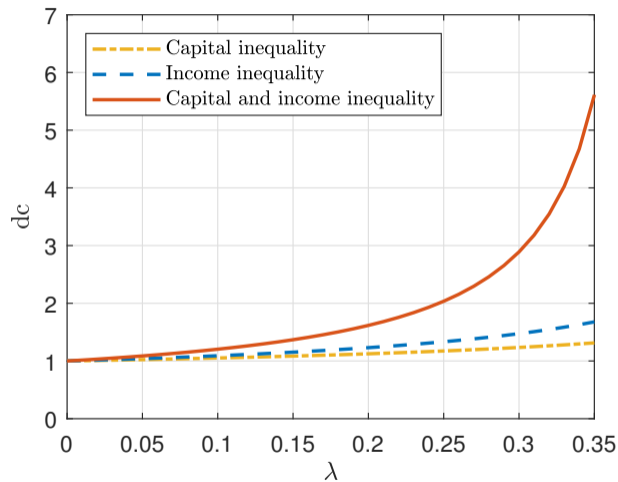
The Multiplier ... of the Multiplier

- Abstracting from direct effect that only $1 - \lambda$ agents react directly to interest rates, the multiplier reads:

$$\frac{1}{1 - \lambda \chi \frac{1-l_Y}{1-\eta l_Y}}$$

- The **income** and **capital** inequality channels compound the aggregate MPC
⇒ The two indirect effects interact non-linearly at each round, multiplying each other
- This is the **multiplier of the multiplier**

A Picture Worth $1/(1-x)$ Words



Multipliers as a function of share of hand-to-mouth λ ($I_Y = 0.235$, $\eta = 2$, and $\chi = 1.75$).

Testable Predictions

1. **Income** and **consumption** inequality:

$$y_t^S - y_t^H = \frac{1 - \chi}{(1 - \lambda)Y_Y^S} y_t$$
$$c_t^S - c_t^H = \frac{1 - \chi C_Y - \eta I_Y}{(1 - \lambda)C_Y} y_t$$

Both are **countercyclical** iff $C_Y(\chi - 1) + I_Y(\eta - 1) > 0$

2. **Consumption** inequality **more countercyclical** than **income** inequality if investment is more than one-to-one procyclical $\eta > 1$

Compare to available evidence (e.g. Coibion et al., 2017) [▶ Details](#)

A Tractable HANK model with Capital

Households

- λ **hand-to-mouth**: only have labor income and consume everything
- $1 - \lambda$ **savers**: get labor, capital and profits income, choose consumption intertemporally
- **Idiosyncratic risk**: household **switch** between types $S \xrightleftharpoons[1-h]{1-s} H$

$$\lambda = \frac{1 - s}{2 - s - h}$$

- **Liquidity**: **liquid** bonds; capital and stocks **illiquid**
- Model matches *micro moments*: iMPCs, income risk etc.

Savers

$$(C_t^S)^{-\frac{1}{\sigma}} = \beta E_t \left\{ \frac{1 + r_t^n}{1 + \pi_{t+1}} \left[s(C_{t+1}^S)^{-\frac{1}{\sigma}} + (1 - s)(C_{t+1}^H)^{-\frac{1}{\sigma}} \right] \right\}$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}^S}{C_t^S} \right)^{-\frac{1}{\sigma}} \left[(1 - \tau^K) R_{t+1}^K + Q_{t+1} \left(1 - \delta + \Phi_{t+1} - \frac{I_{t+1}}{K_{t+1}} \Phi'_{t+1} \right) \right] \right\}$$

with $Q_t = \left(\Phi' \left(\frac{I_t}{K_t} \right) \right)^{-1}$.

Hand-to-mouth

$$C_t^H = \frac{W_t}{P_t} N_t^H + T_t^H$$

Firm Behavior

- Phillips curve: $\pi_t = \beta E_t \pi_{t+1} + \psi mc_t$

- Factor prices:
$$\frac{K_t}{N_t} = \frac{\alpha}{1-\alpha} \frac{W_t}{P_t R_t^K}$$

$$\frac{MC_t}{P_t} = (1-\alpha)^{\alpha-1} \alpha^{-\alpha} \left(R_t^K\right)^\alpha \left(\frac{W_t}{P_t}\right)^{1-\alpha}$$

Government

- Monetary policy: $r_t^n = \phi_\pi \pi_t + \varepsilon_t$
- Fiscal **redistribution**: $\lambda T_{H,t} = \tau^D D_t + \tau^K r_t^K K_t$
 - **No income inequality** $\chi = 1$ under perfect redistribution with $\tau^D = \tau^K = \lambda$

Parameterization

Parameter	Value	Description
α	0.33	Capital share of output
δ	0.025	Depreciation rate per quarter
ω	10	Elasticity of investment to Q
β	0.99	Discount factor
s	1 / 0.98	Probability of staying unconstrained
σ	1	Intertemporal elasticity of substitution
$1/\varphi$	1.00	Frisch elasticity
λ	0 / 0.27	Share of hand-to-mouth
τ^D, τ^K	$= \begin{cases} 0 & \text{no redistribution} \\ \lambda & \text{full redistribution} \end{cases}$	Taxes on profits and capital
ψ	0.050	Slope of PC
ψ_w	∞ / 0.075	Slope of PC wages
ϕ_π	1.50	Taylor rule coefficient
ϕ_i	0.00	Interest rate smoothing
ρ_i	0.60	Persistence MP shock

Quantifying the Complementarity

Amplification of the Monetary Policy Effects on Consumption

	Rep. agent	Heterogeneous agents	
		Prop. incomes	Inequality
No capital	1.00	1.00	1.51
Capital	0.66	1.11	2.25

- Dampening in RANK with capital (real rate channel)

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Quantifying the Complementarity

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No capital	1.00	1.00	1.51	1.60
Capital	0.66	1.11	2.25	2.62

- Dampening in RANK with capital (real rate channel)
- **Capital inequality** amplifies even under no income inequality
- **Income inequality** can also lead to substantial amplification
- Channels are strongly **complementary**: $2.25 \gg 1.11 * 1.51$
- Idiosyncratic risk **reinforces** complementarity

Analytical Solution

- **Novel analytical solution:** (under $\delta = 1$, $\omega = \infty$, $\sigma = 1$, $\varphi = 0$, $s = 1$, cont. PC and $\phi_\pi = 1$)

$$\frac{\partial c_t}{\partial (-\varepsilon_t)} = \frac{1 - \lambda}{1 - \lambda \chi_K} \left\{ 1 + \frac{\lambda \alpha \beta}{(1 - \lambda)(1 - \alpha \beta)} \frac{(1 - \alpha)}{1 + \alpha \frac{\lambda(\chi_K - 1)}{1 - \lambda \chi_K} \left(1 + \frac{1 - \alpha \beta}{1 - \alpha}\right)} \right\}$$

with $\chi_K = 1 + \frac{1 - \alpha}{1 - \alpha \beta}$.

- **Full-redistribution**

$$\frac{\partial c_t}{\partial (-\varepsilon_t)} = 1 + \frac{(1 - \alpha) \lambda \alpha \beta}{(1 - \lambda)(1 - \alpha \beta)}$$

- **No capital**

$$\frac{\partial c_t}{\partial (-\varepsilon_t)} = \frac{1 - \lambda}{1 - \lambda \chi_{noK}},$$

with $\chi_{noK} = 2 - \alpha$.

⇒ **Consistent** with simple model [▶ Details](#)

Role of Redistribution

Redistribution	Profit income	
	Yes	No
Capital income	Yes	1.15
	No	0.50

- Profits are countercyclical $\rightarrow Y_H$ less cyclical \rightarrow dampening
- Capital income strongly procyclical $\rightarrow Y_H$ more cyclical \rightarrow **Amplification**

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Capital income	Yes	1.15
	No	0.50

- Profits are countercyclical $\rightarrow Y_H$ less cyclical \rightarrow dampening
- Capital income strongly procyclical $\rightarrow Y_H$ more cyclical \rightarrow **Amplification**
- Less stark with sticky wages as profits less countercyclical

Amplification of the Monetary Policy Effects on Consumption

	Rep. agent	Heterogeneous agents		
		Prop. incomes	Inequality	Inequality and risk
No capital	1.00	1.00	1.01	1.02
Capital	0.94	1.53	1.77	1.95

- Amplifies **capital inequality** channel

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- Amplifies **capital inequality** channel
- Dampens **income inequality** channel

Amplification of the Monetary Policy Effects on Consumption

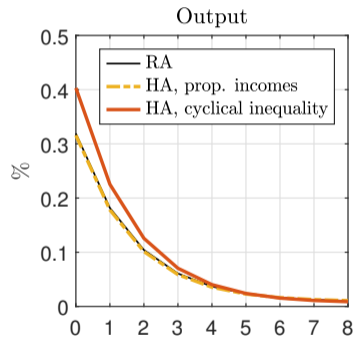
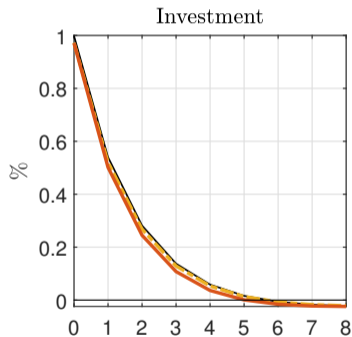
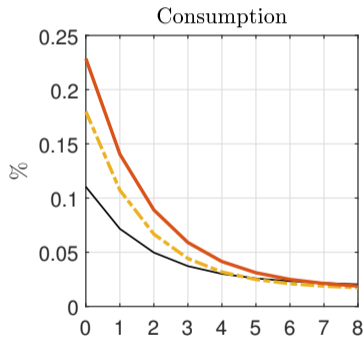
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- Amplifies **capital inequality** channel
- Dampens **income inequality** channel
- **Complementarity** robust

- Further step towards **Macro convergence**
 - Bring capital back to policy-relevant, monetary models
 - Bridge quantitative and tractable HANK models
- Complementarity of **capital** and **income inequality** channel
 - Through a **multiplier of the multiplier**
- Key for monetary policy *what income* is redistributed
 - Optimal policy?

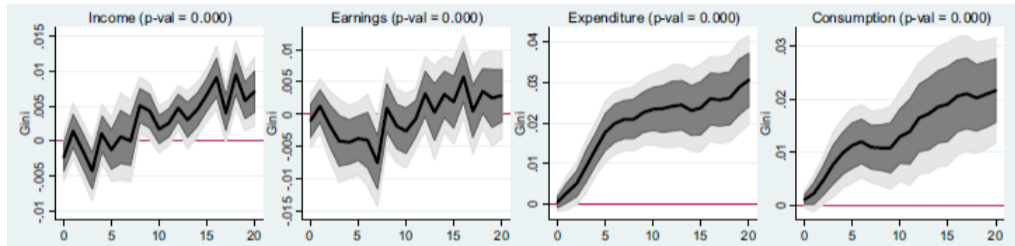
Thank you!

Aggregate responses



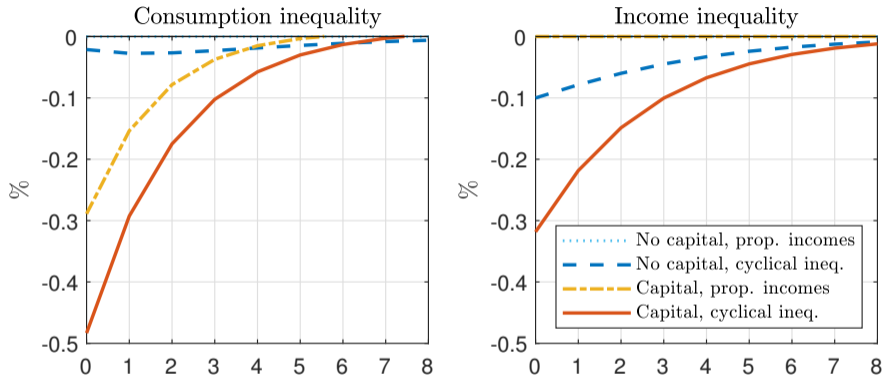
Testable Predictions

Distributional effects (Coibion et al., 2017)



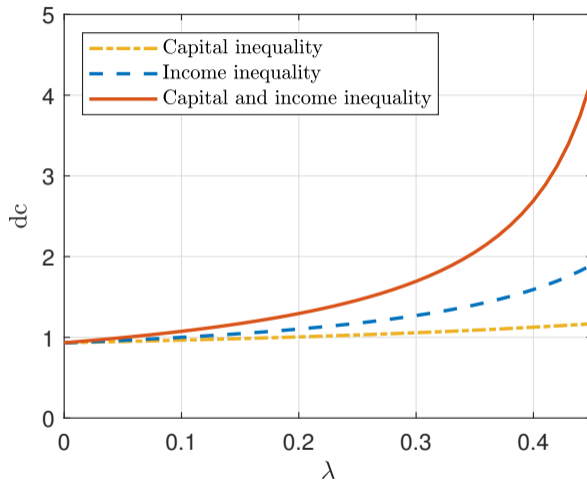
- Consumption and income inequality are **countercyclical** with
- Consumption inequality *more so*

Testable Predictions



- Only model with **capital** and **income inequality** can generate this

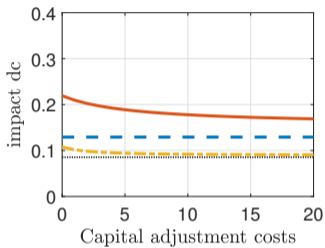
Amplification in analytically tractable case



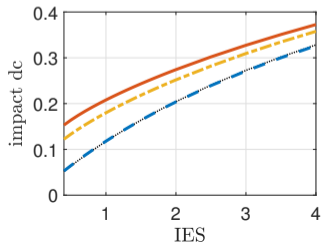
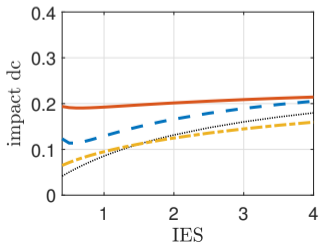
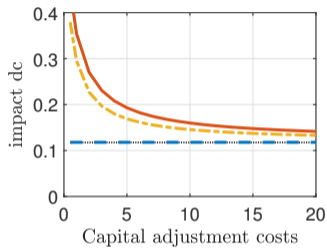
Multipliers as a function of share of hand-to-mouth λ .

Robustness

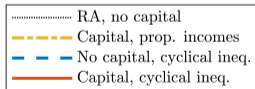
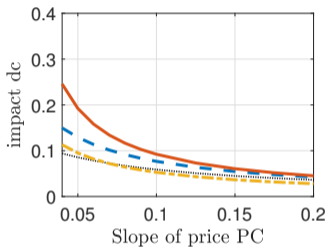
(A) Flexible wages



(B) Sticky wages



(A) Flexible wages



(B) Sticky wages

